

# Mathematics!



## **A Story of Units Parent Handbook**

**Grade 3  
Module 5**

# Fractions as Numbers on a Number Line

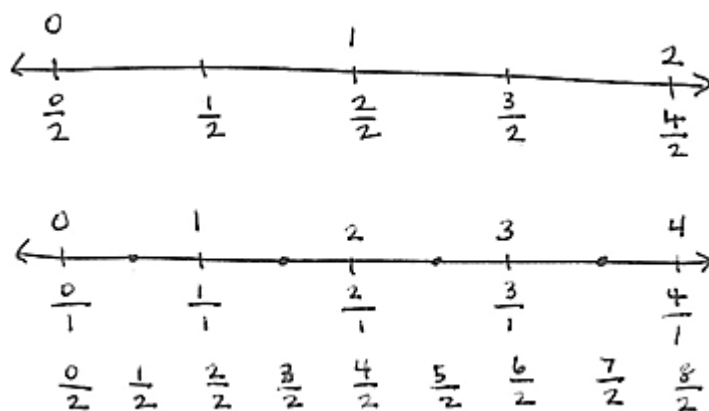
## Line

### OVERVIEW

In this 35-day Grade 3 module, students extend and deepen Grade 2 practice with equal shares to understanding fractions as equal partitions of a whole. Their knowledge becomes more formal as they work with area models and the number line.

Topic A opens Module 5 with students actively partitioning different models of wholes into equal parts (e.g., concrete models, fraction strips, and drawn pictorial area models on paper). They identify and count equal parts as *1 half*, *1 fourth*, *1 third*, *1 sixth*, and *1 eighth* in unit form before an introduction to the unit fraction  $\frac{1}{b}$ . In Topic B, students compare unit fractions and learn to build non-unit fractions with unit fractions as basic building blocks. This parallels the understanding that the number 1 is the basic building block of whole numbers. In Topic C, students practice comparing unit fractions with fraction strips, specifying the whole and labeling fractions in relation to the number of equal parts in that whole.

Students transfer their work to the number line in Topic D. They begin by using the interval from 0 to 1 as the whole. Continuing beyond the first interval, they partition, place, count, and compare fractions on the number line. In Topic E, they notice that some fractions with different units are placed at the exact same point on the number line, and therefore are equal. For example,  $\frac{1}{2}$ ,  $\frac{2}{4}$ ,  $\frac{3}{6}$ , and  $\frac{4}{8}$  are equivalent fractions. Students recognize that whole numbers can be written as fractions, as exemplified on the number lines below.



Topic F concludes the module with comparing fractions that have the same numerator. As they compare fractions by reasoning about their size, students understand that fractions with the same numerator and a larger denominator are actually smaller pieces of the whole. Topic F leaves students with a new method for precisely partitioning a number line into unit fractions of any size without using a ruler.

# Terminology

## New or Recently Introduced Terms

- Unit fraction (fractions with numerator 1)
- Non-unit fraction (fractions with numerators other than 1)
- Fractional unit (half, third, fourth, etc.)
- Equal parts (parts with equal measurements)
- Unit interval (the interval from 0 to 1, measured by length)
- Equivalent fraction (2 fractions that name the same size)
- Copies (refers to the number of unit fractions in 1 whole)

## Familiar Terms and Symbols

- Number line
- Arrays
- Halves, thirds, fourths, sixths, eighths ( $1/2$ ,  $1/3$ ,  $1/4$ ,  $1/6$ ,  $1/8$ )
- Half of, one third of, one fourth of, etc. ( $1/2$ ,  $1/3$ ,  $1/4$ ,  $1/6$ ,  $1/8$ )
- =, <, > (equal, less than, greater than)
- Equal shares (pieces of a whole that are the same size)
- Whole (e.g., 2 halves, 3 thirds, etc.)
- Fraction (e.g.,  $1/3$ ,  $2/3$ ,  $3/3$ ,  $4/3$ )
- Partition (divide a whole into equal parts)

## Suggested Tools and Representations

- Number line
- Tape diagram
- Arrays
- Concrete area models (e.g., water, string, clay)
- Pictorial area model (e.g., drawing of a circle or square)
- Fraction strips (made from paper, used to fold and model parts of a whole)

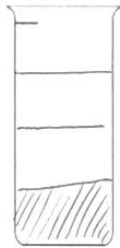
# Lesson 1

Objective: Specify and partition a whole into equal parts, identifying and counting unit fractions using concrete models.

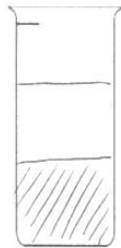
1. A beaker is considered full when the liquid reaches the fill line shown near the top. Estimate the amount of water in the beaker by shading the drawing as indicated. The first one is done for you.



1 half

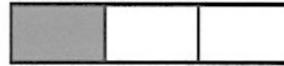


1 fourth



1 third

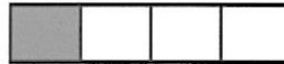
2. Juanita cut her string cheese into equal pieces as shown in the rectangles below. In the blanks below, name the fraction of the string cheese represented by the shaded part.



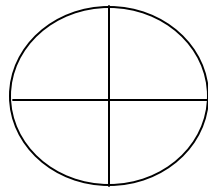
1 third



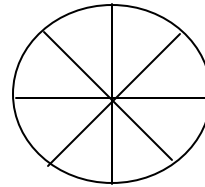
1 sixth



1 fourth



Fourths



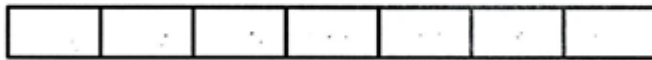
Eighths

# Lesson 2

Objective: Specify and partition a whole into equal parts, identifying and counting unit fractions by folding fraction strips.

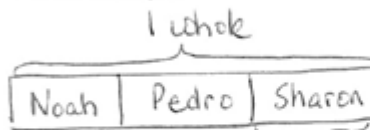


c) There are 7 equal parts in all. 3 are shaded.



d) There are 7 equal parts in all. 0 are shaded.

Noah, Pedro and Sharon want to share a whole candy bar fairly. Which of your fraction strips shows how they can each get an equal part? Draw the candy bar below. Label to show who gets which part. Label the fraction of the candy bar Sharon gets.

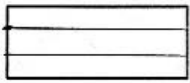


Sharon gets 1 third of the candy bar.

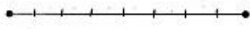
### Lesson 3

Objective: Specify and partition a whole into equal parts, identifying and counting unit fractions by drawing pictorial area models.

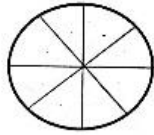
Each shape is 1 whole. Estimate to divide each into equal parts (do not draw fourths). Divide each whole using a different fractional unit. Write the name of the fractional unit on the line below the shape.



1 third

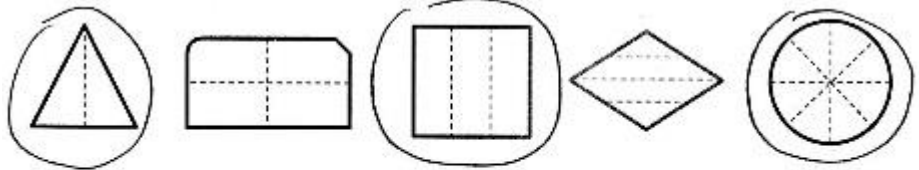


1 eighth



1 eighth

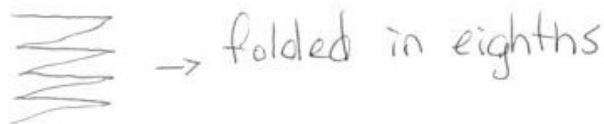
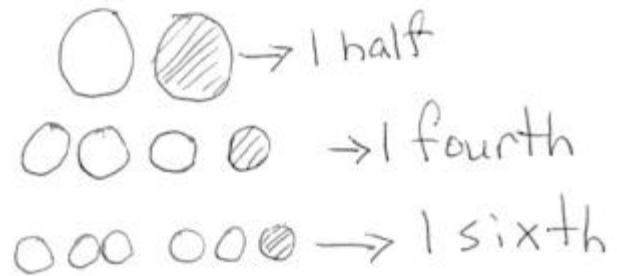
Circle the shapes that are divided into equal parts. Write a sentence telling what "equal parts" means.



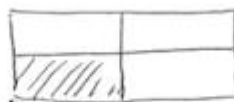
Equal parts means that the pieces are the same size and the same shape.

### Lesson 4

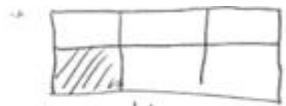
Objective: Represent and identify fractional parts of different wholes.



1 half




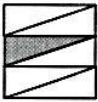
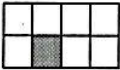
1 fourth



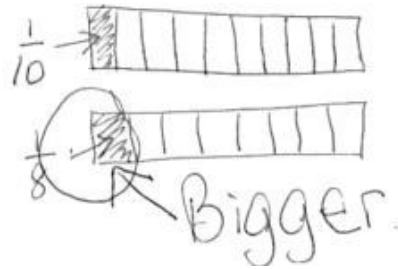
1 sixth

## Lesson 5

Objective: Partition a whole into equal parts and define the equal parts to identify the unit fraction numerically.

	5	1	$\frac{1}{5}$ fifth	$\frac{1}{5}$
	6	1	$\frac{1}{6}$ sixth	$\frac{1}{6}$
	8	1	$\frac{1}{8}$ eighth	$\frac{1}{8}$

Andre thinks it's strange that  $\frac{1}{10}$  of the cake would be less than  $\frac{1}{8}$  of the cake, since ten is bigger than eight. To explain to Andre, draw 2 identical rectangles to stand for the cakes. Show 1 tenth shaded on one and 1 eighth shaded on the other. Label the unit fractions and show him which slice is bigger.



## Lesson 6

Objective: Build non-unit fractions less than one whole from unit fractions.

$$2 \text{ thirds} = \frac{2}{3}$$



Mr. Stevens bought 8 liters of soda for a party. His friends drank 1 liter.

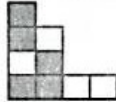


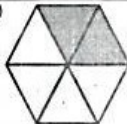
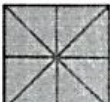
a) What fraction of the soda did his guests drink?



They drank  $\frac{1}{8}$  of the soda.

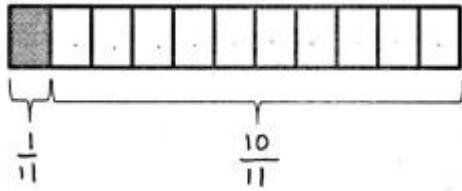
b) What fraction of the soda was left?

$\frac{7}{8}$  of the soda were left.

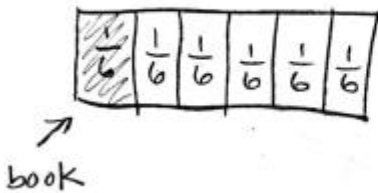
a) 	9	5	$\frac{1}{9}$	$\frac{5}{9}$
b) 	7	3	$\frac{1}{7}$	$\frac{3}{7}$
c) 	5	4	$\frac{1}{5}$	$\frac{4}{5}$
d) 	6	2	$\frac{1}{6}$	$\frac{2}{6}$
e) 	8	8	$\frac{1}{8}$	$\frac{8}{8}$

## Lesson 7

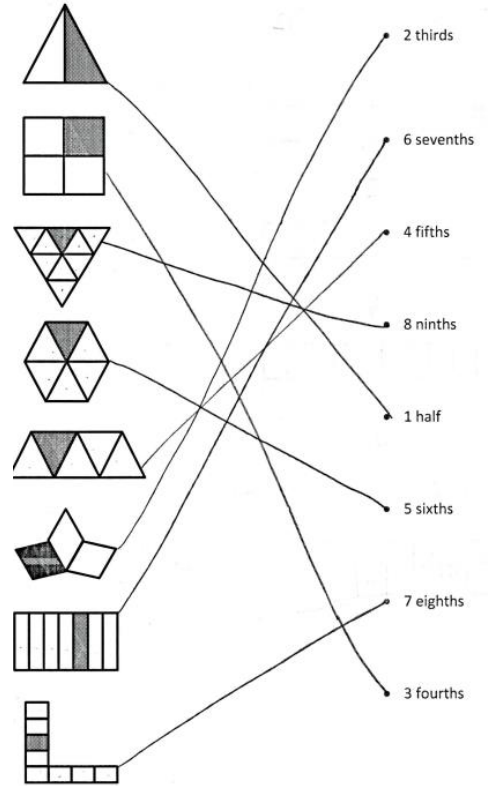
Objective: Identify and represent shaded and non-shaded parts of one whole as fractions.



Avanti read  $\frac{1}{6}$  of her book. What fraction of the book has she not read yet?

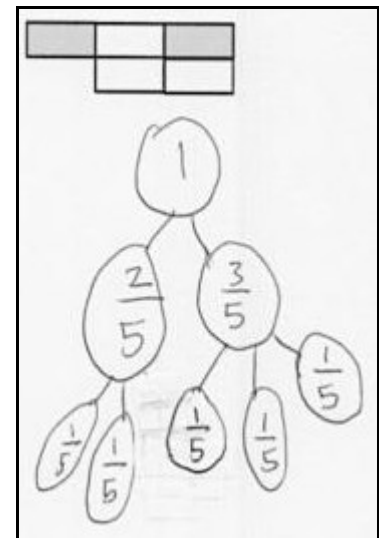
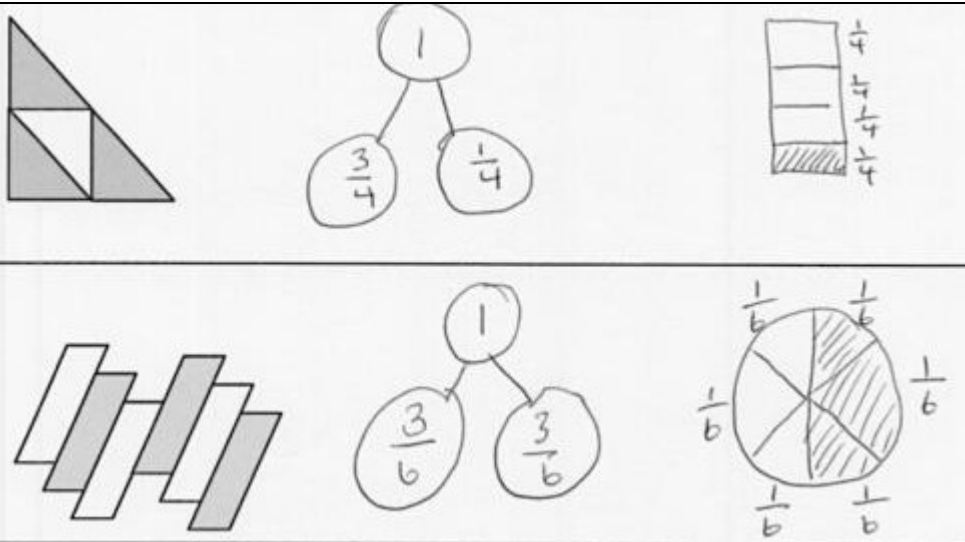


She has not read  $\frac{5}{6}$  of her book



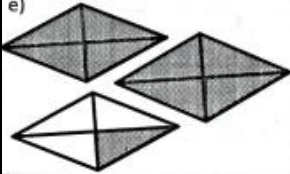

## Lesson 8

Objective: Represent parts of one whole as fractions with number bonds.



# Lesson 9

Objective: Build and write fractions greater than one whole using unit fractions.

e)		$\frac{1}{4}$	9	$\frac{9}{4}$
f)		$\frac{1}{3}$	7	$\frac{7}{3}$

Mrs. Jawlik baked 2 pans of brownies. Draw the pans and estimate to partition each pan into 8 equal pieces.

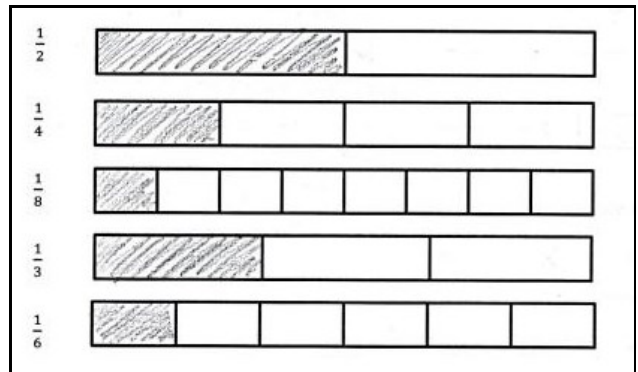


a) Mrs. Jawlik's children gobble up 10 pieces. Shade the amount that was eaten.

b) Write a fraction to show how many pans of brownies her children ate.  $\frac{10}{8}$  brownies. OR  
1 pan and  $\frac{2}{8}$  more.

# Lesson 10

Objective: Compare unit fractions by reasoning about their size using fraction strips.



a)  $\frac{1}{2}$  is greater than  $\frac{1}{4}$

b)  $\frac{1}{6}$  is less than  $\frac{1}{2}$

d)  $\frac{1}{3}$  is greater than  $\frac{1}{6}$

Your friend Eric says that  $\frac{1}{6}$  is greater than  $\frac{1}{5}$  because 6 is greater than 5. Is Eric correct?

Use words and pictures to explain what happens to the size of a unit fraction when the number of parts gets larger.

He is wrong. Because if you have 1 whole and you make 6 pieces then each piece is smaller than if you only have 5 pieces. Like Lily and her water and oil.



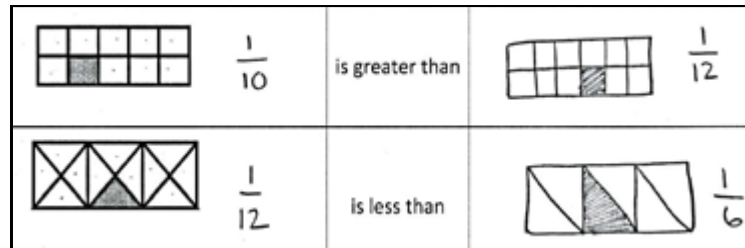
5ths are bigger because when the number of parts is smaller the pieces are bigger.



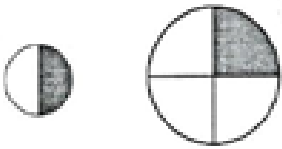
## Lesson 11

Objective: Compare unit fractions with different sized models representing the whole.

When comparing unit fractions where the wholes are the same size we can see larger denominators make smaller unit fractions (portions).



You cannot compare fractions when the wholes are different sizes.



Usually  $\frac{1}{2}$  is larger than  $\frac{1}{4}$ . But here the circles are not the same size so the fractions can't be compared proportionally.

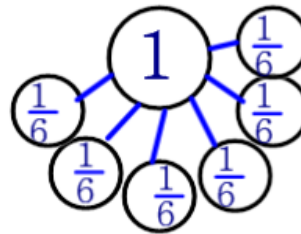
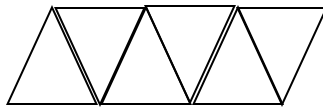
Another example is that  $\frac{1}{2}$  of a small pizza is not the same as  $\frac{1}{2}$  of a large pizza.

## Lesson 12

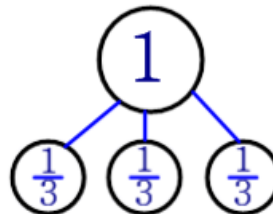
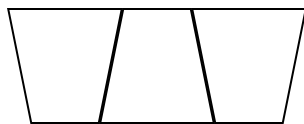
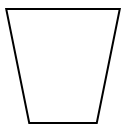
Objective: Specify the corresponding whole when presented with one equal part.

Each shape represents the given unit fraction. Estimate to draw the whole.

$\frac{1}{6}$


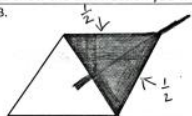
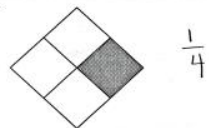
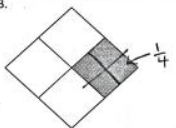


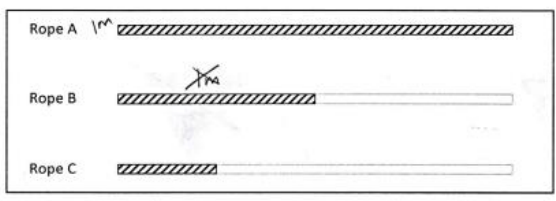
$\frac{1}{3}$



# Lesson 13

Objective: Identify a shaded fractional part in different ways depending on the designation of the whole.

<p>The shape represents 1 whole. Write a fraction to describe the shaded part.</p> <p>1A.</p> 	<p>The shaded part represents 1 whole. Divide 1 whole to show the unit fraction you wrote in A.</p> <p>B.</p> 
<p>2A.</p> 	<p>B.</p> 



Rope A  $\frac{1}{2}$

Rope B  $\frac{1}{2}$

Rope C  $\frac{1}{4}$

a. Rope C is  $\frac{1}{2}$  of Rope B.

b. Rope B is  $\frac{1}{2}$  of Rope A.

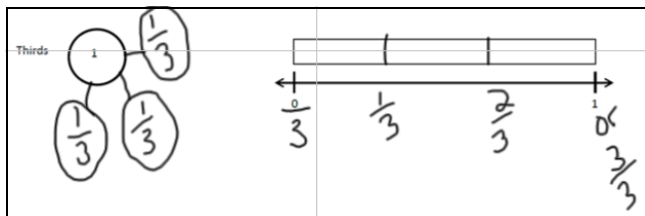
c. Rope B is  $\frac{1}{2}$  of Rope A.

d. Rope C is  $\frac{1}{4}$  of Rope A.

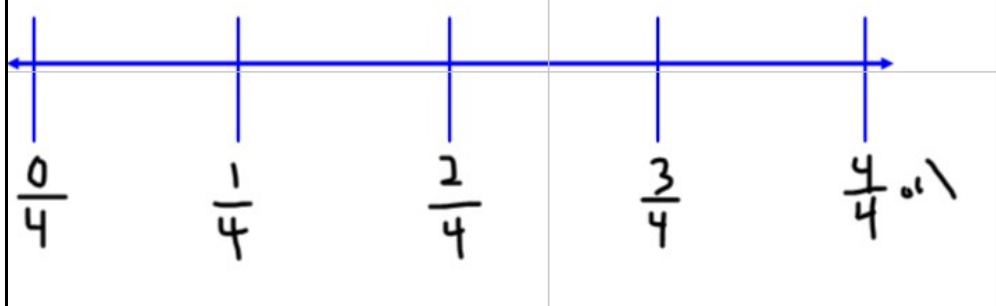
e. If Rope B measures 1m long, then Rope A is 2 m long and Rope C is  $\frac{1}{2}$  m long.

# Lesson 14

Objective: Place unit fractions on a number line with endpoints 0 and 1.



Trevor needs to let his puppy outside every quarter (1 fourth) hour to potty train him. Draw and label a number line from 0 hours to 1 hour to show every 1 fourth hour. Include 0 fourths and 4 fourths hour. Label 0 hours and 1 hour, too.

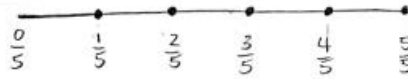


# Lesson 15

Objective: Place any fraction on a number line with endpoints 0 and 1.

For his boat, James stretched out a rope with 5 equally spaced knots.

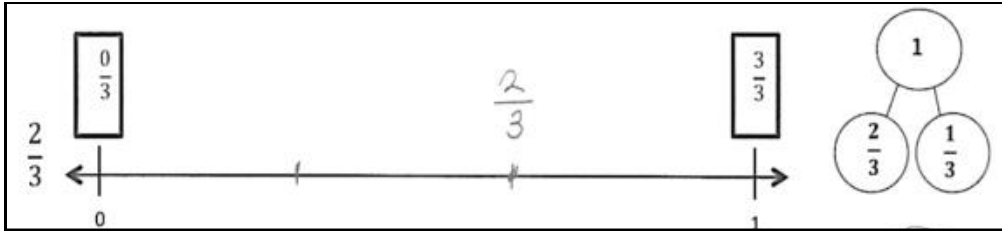
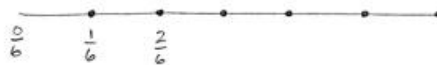
a. Draw his rope.



b. Starting at the first knot and ending at the last knot, how many unit fractions are formed by the 5 knots? Label each unit fraction at the knot. *5 unit fractions*

c. What fraction of the rope is labeled at the third knot? *3/5*

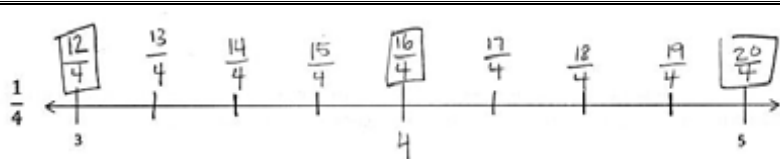
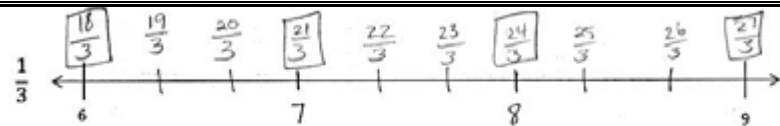
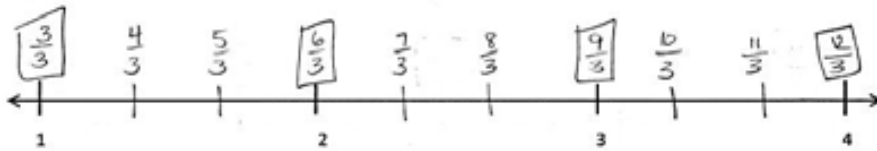
d. What if the rope had 6 equally spaced knots? What fraction of the rope would be measured by the first 2 knots? *2/6*



# Lesson 16

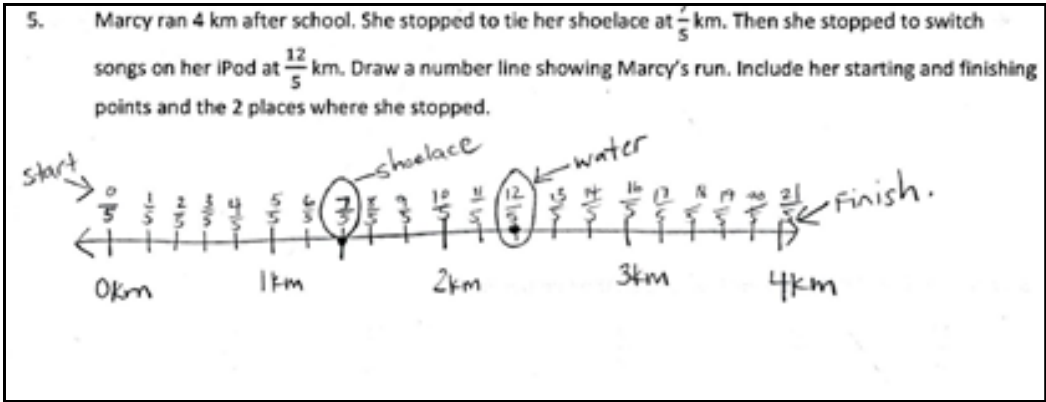
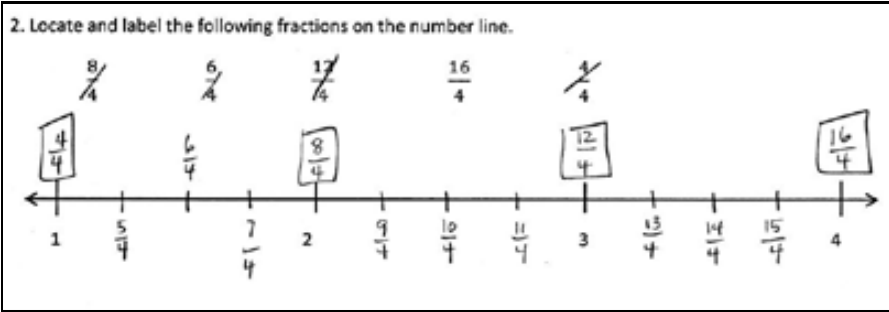
Objective: Place whole number fractions and unit fractions between whole numbers on the number line.

3. Partition each whole into 3 unit fractions. Label each unit fraction. Count up as you go. Box the fractions that are located at the same points as whole numbers.



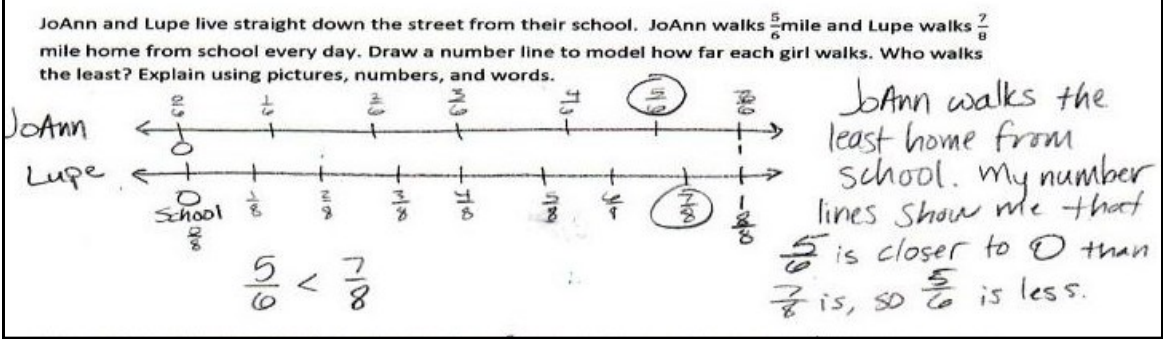
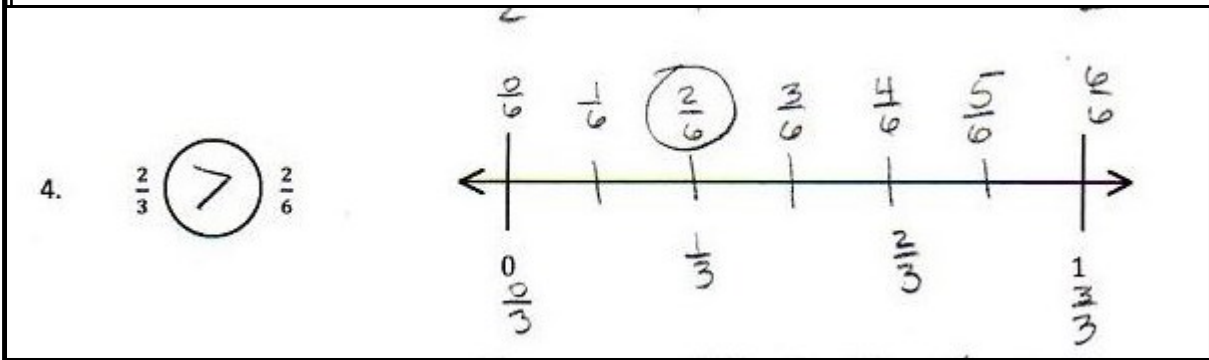
# Lesson 17

Objective: Practice placing various fractions on the number line.



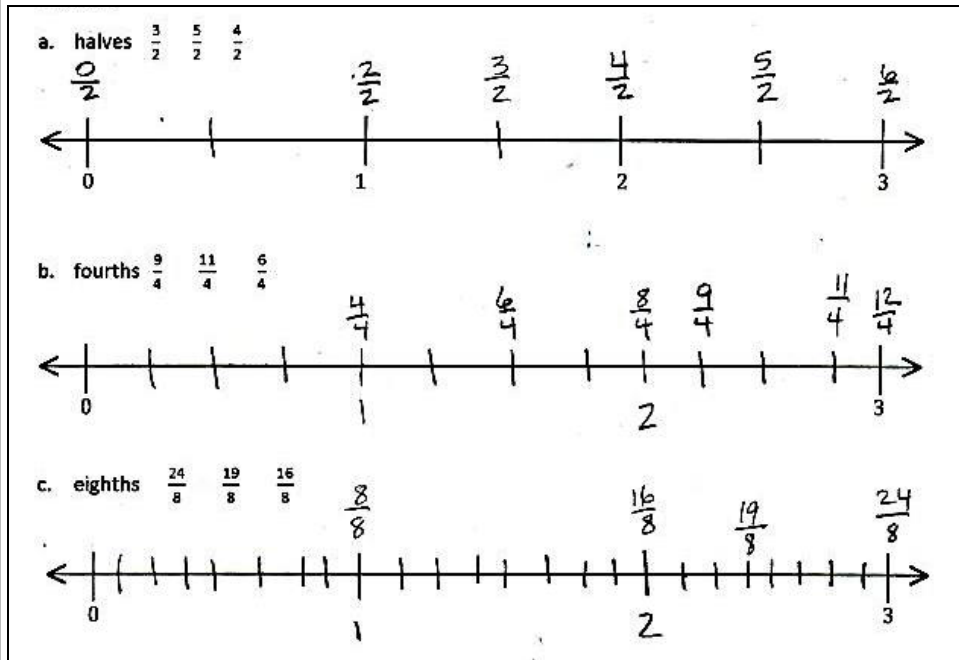
# Lesson 18

Objective: Compare fractions and whole numbers on the number line by reasoning about their distance from 0.



# Lesson 19

Objective: Understand distance and position on the number line as strategies for comparing fractions.

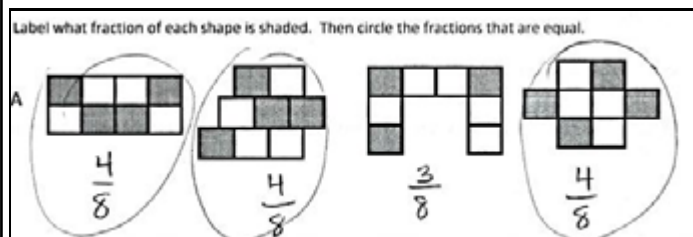


$$\frac{3}{2} < \frac{5}{2}$$

$$\frac{6}{4} < \frac{16}{8}$$

# Lesson 20

Objective: Recognize and show that equivalent fractions have the same size, though not necessarily the same shape.



3. Ann has 6 small square pieces of paper. 2 squares are grey. Ann cuts the 2 grey squares in half with a diagonal line from one corner to the other.

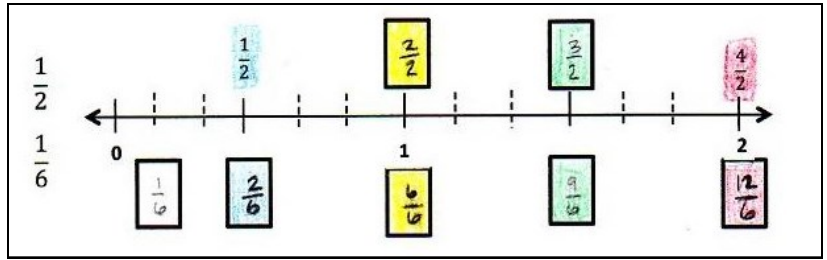
a) What shapes does she have now?  
*triangles and squares*

b) How many of each shape does she have?  
*4 triangles and 4 squares*

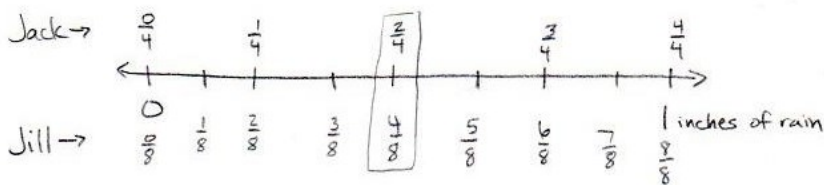
c) Use all the shapes with no overlaps. Draw different ways Ann's set of shapes might look. What fraction of all the shapes is grey?  
 *$\frac{2}{6}$  are grey.*

## Lesson 21

Objective: Recognize and show that equivalent fractions refer to the same point on the number line.



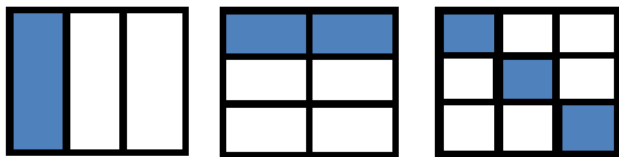
Jack and Jill use rain gauges the same size and shape to measure rain on the top of a hill. Jack uses a rain gauge marked in fourths of an inch. Jill's gauge measures rain in eighths of an inch. On Thursday, Jack's gauge measured inches of rain. They both had the same amount of water, so what was the reading on Jill's gauge Thursday? Draw a number line to help explain your thinking.



Jill's reading was  $\frac{4}{8}$  inch of rain because  $\frac{2}{4} = \frac{4}{8}$ .

## Lesson 22

Objective: Generate simple equivalent fractions by using visual fraction models and the number line.



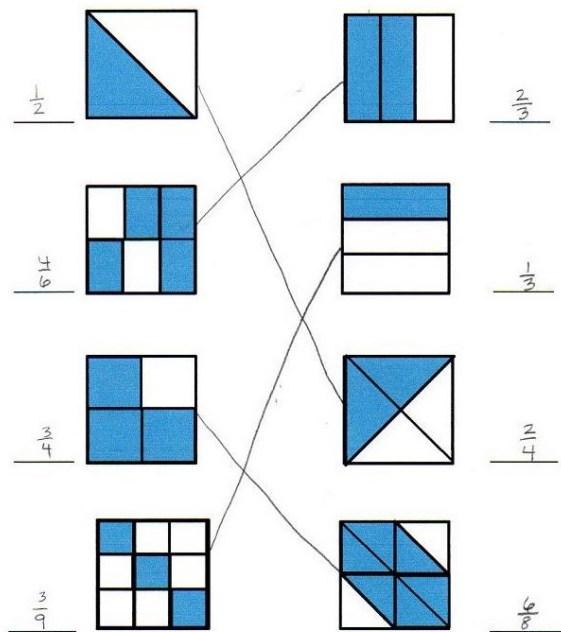
$\frac{1}{3}$

$\frac{2}{6}$

$\frac{3}{9}$

All of these fractions are equivalent. They take up the same portion of the whole.

1. Write what fraction of the square is shaded in the blanks then match the equivalent fractions.



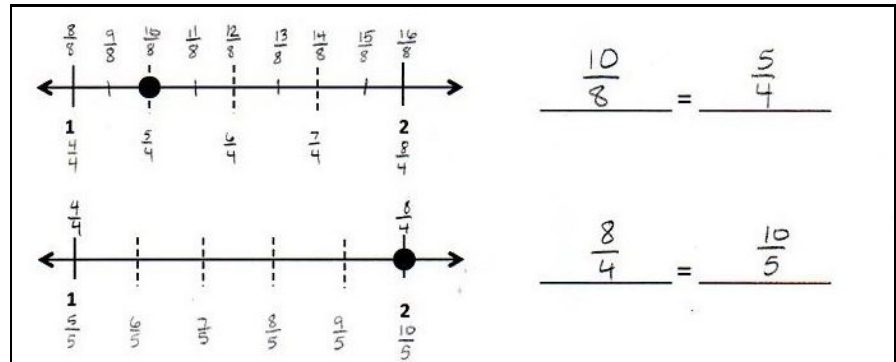
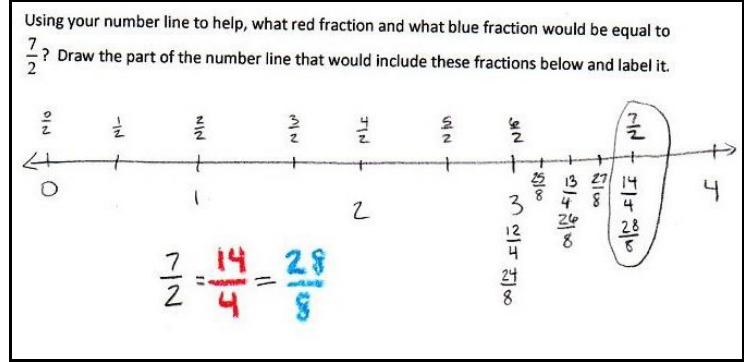
How many sixths does it take to make the same amount as  $\frac{1}{3}$ ? Explain your answer in words and pictures.



2 sixths because sixths are twice as many as thirds so you need twice as many copies.

## Lesson 23

Objective: Generate simple equivalent fractions by using visual fraction models and the number line.

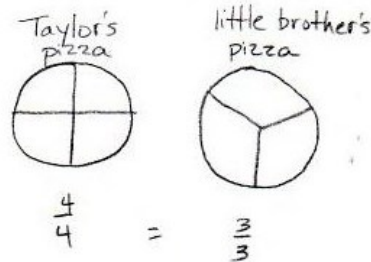


## Lesson 24

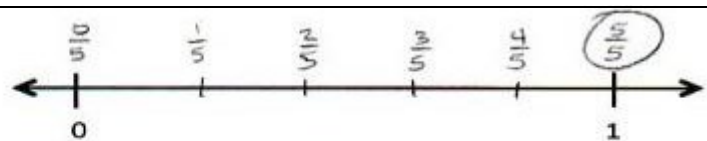
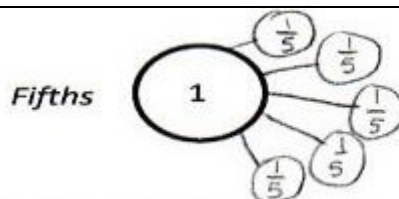
Objective: Express whole numbers as fractions and recognize equivalence with different units.

Taylor took his little brother to get pizza. Each boy ordered a small pizza. Taylor's pizza was cut in fourths and his brother's was cut in thirds. After they had both eaten all of their pizza, Taylor's little brother said, "Hey, that was no fair! You got more than me! You got 4 pieces, I only got 3!"

Should Taylor's little brother be mad? What could you say to explain the situation to him? Use words, pictures or a number line.

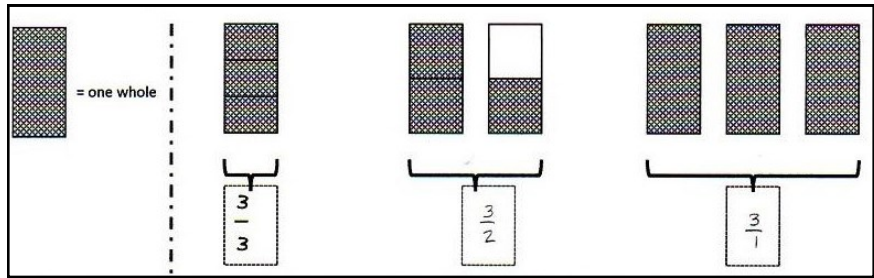


4 fourths is the same as 3 thirds because both are equal to 1. The pizzas were the same size so it didn't matter how many slices each one had. I would show his brother both of their empty pizza pans so the brother could see that the pans started as the same size, but were cut differently.



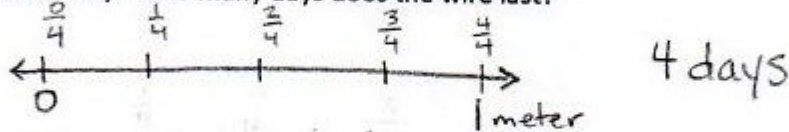
## Lesson 25

Objective: Express whole number fractions on the number line when the unit interval is 1.



Sammy uses  $\frac{1}{4}$  meter of wire each day to make things.

a) Draw a number line to represent 1 meter of wire. Partition the number line to represent how much Sammy uses each day. How many days does the wire last?



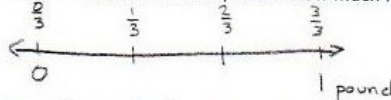
b) How many days will 3 meters of wire last?



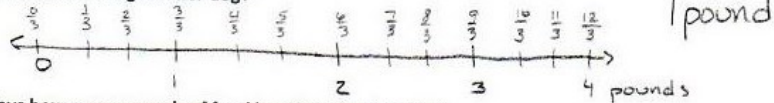
## Lesson 26

Objective: Decompose whole number fractions greater than 1 using whole number equivalence with various models.

Cindy feeds her dog  $\frac{1}{3}$  pound of food each day. Draw a number line to represent 1 pound of food. Partition the number line to represent how much food she uses each day.



a) Draw another number line to represent 4 pounds of food. After 3 days, how many pounds of food has she given her dog?

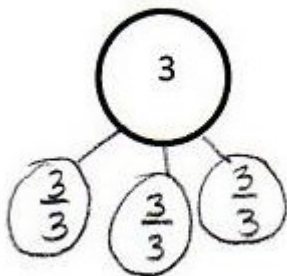


b) After 6 days how many pounds of food has she given her dog?

2 pounds

$$3 = \frac{9}{3} \text{ thirds}$$

$$3 = \frac{9}{3}$$




Write the fraction that names the whole numbers for each unit fraction. The first one has been done.

	2	3	4
halves	$\frac{4}{2}$	$\frac{6}{2}$	$\frac{8}{2}$
thirds	$\frac{6}{3}$	$\frac{9}{3}$	$\frac{12}{3}$
fourths	$\frac{8}{4}$	$\frac{12}{4}$	$\frac{16}{4}$
sixths	$\frac{12}{6}$	$\frac{18}{6}$	$\frac{24}{6}$



## Lesson 27

Objective: Explain equivalence by manipulating units and reasoning about their size.



1 half is equal to 4 eighths

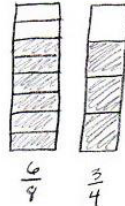
$$\frac{1}{2} = \frac{4}{8}$$

The whole stays the same.

What happened to the size of the equal parts when there were more equal parts?  
They got smaller.

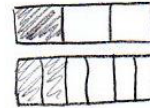
What happened to the number of equal parts when the equal parts became smaller?  
You needed more.

When the whole is the same, why does it take 6 copies of 1 eighth to show 3 copies of 1 fourth? Draw a model to support your answer.



The unit "eighths" is smaller than the unit "fourths" so you need more copies of the smaller unit to be equal to the larger unit.

When the whole is the same, how many sixths does it take to make 1 third? Draw a model to support your answer.

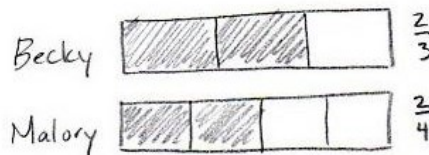


2 sixths or  $\frac{2}{6} = \frac{1}{3}$

## Lesson 28

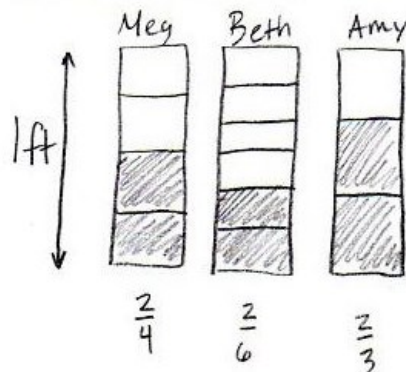
Objective: Compare fractions with the same numerator pictorially.

7. Becky and her twin sister, Malory, each got matching piggy banks for their birthday. Becky filled  $\frac{2}{3}$  of her piggy bank with pennies. Malory, filled her piggy bank  $\frac{2}{4}$  full of pennies. Whose piggy bank has more pennies? Use a tape diagram to show your work.



Becky's piggy bank has more pennies.

Heidi's little sister was comparing the height of her dolls. Dolly Meg is  $\frac{2}{4}$  foot tall, Dolly Beth is  $\frac{2}{6}$  foot tall, and Dolly Amy is  $\frac{2}{3}$  foot tall. After measuring the dolls, her sister lined them up, shortest to tallest. Compare the height of the dolls to place them in order from shortest to tallest. Draw a picture to support your answer.

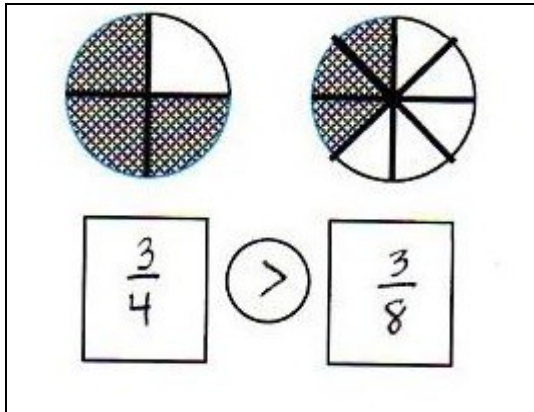
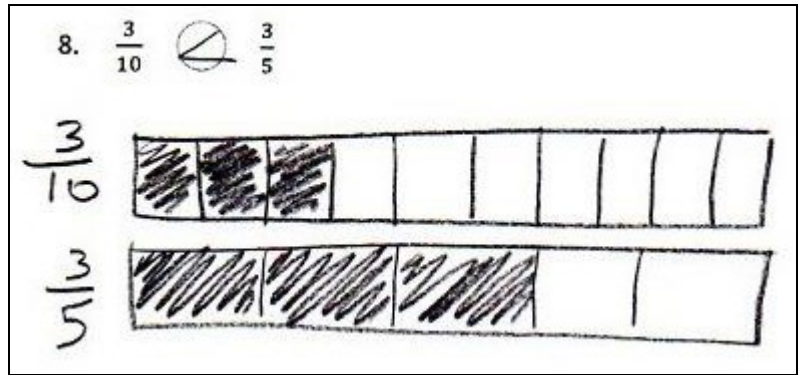


shortest  $\longrightarrow$  tallest  
Beth  $\rightarrow$  Meg  $\rightarrow$  Amy

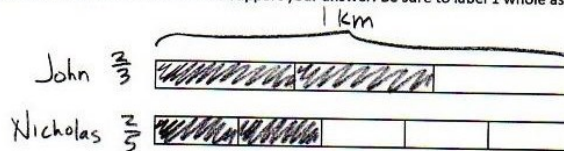
The shortest doll is Beth, then Meg, and the tallest doll is Amy.

## Lesson 29

Objective: Compare fractions with the same numerator using  $<$ ,  $>$ , or  $=$  and use a model to reason about their size.



John ran  $\frac{2}{3}$  kilometer after school. Nicholas ran  $\frac{2}{5}$  kilometer after school. Who ran the shorter distance? Use the model below to support your answer. Be sure to label 1 whole as 1 kilometer.



Nicholas ran the shorter distance. Even though they both ran 2 of their units, fifths are a smaller unit than thirds, so  $\frac{2}{5}$  is smaller (or shorter) than  $\frac{2}{3}$ .

Even though this module concentrates on fractions it is still important to continue practicing multiplication and division facts and to review addition and subtraction facts. Quick 5-10 minute activities are essential for memorization. Here are some ways to assist your child with memorizing basic facts:

- Flash Cards
  - ◇ both you and your child should say the fact aloud
  - ◇ begin learning them in order
- Skip counting up and down. Try beginning at different starting points.
  - ◇ ie: 3, 6, 9, 12– 9, 6, 3      16, 20, 24, 28, 32–28, 24, 20, 16
- Have quick routine math talks in the car, store, and anywhere that seems appropriate.
- Computer Aides such as [xtramath.org](http://xtramath.org)